The Heating Effect of the Currents in Precise Measurements of Electrical Resistance.

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(From the National Physical Laboratory.)

In a paper on the specific heat of water,* one of us describes an experiment in which the passage of a current of 4·4 ampères through an oil-cooled manganin resistance of 1·2-mm. wire produced an increase in the resistance corresponding to an increase of temperature of 60° C. As the cooling surface of the resistance was 16 sq. cm. per watt, such a large increase of temperature was thought to be improbable, and the change of resistance was attributed to some other cause. However, the effect of the passage of the current was similar to that resulting on raising the temperature, and, because of this, it was proposed to call the change a *thermoid* effect. With platinum wires similar results were obtained, the increase of resistance being nearly proportional to the square of the current and inversely proportional to the radius of the wire.

The change of resistance with varying current was investigated by Dr. E. H. Griffiths in 1893.† Using a fine platinum wire, he found the increase of resistance to be proportional to the square of the applied voltage. With regard to this increase, Dr. Griffiths remarks:—"It seemed absolutely immaterial whether the current was on for only a few seconds or indefinitely." Previous experiments on an alloy of copper, manganese, and nickel had shown that the resistance of the alloy changed appreciably in the course of one and a-half hours when a current of $\frac{1}{2}$ ampère was passed through it, and Dr. Griffiths states that the rise appeared to be a function of the current rather than of the time. If, in Dr. Griffiths' determination of the mechanical equivalent of heat, he had neglected the rise of resistance of the wire with increasing current, an error of about 1 part in 60 would have been introduced.

The question as to whether the rise of resistance is due entirely to increase of temperature or due in part to some other effect of the current is of extreme importance for precise electrical measurements. Moreover, since in many measurements it is necessary to use large currents, it is desirable to

* "The Specific Heat of Water and the Mechanical Equivalent of the Calorie at Temperatures from 0° C. to 80° C.," 'Phil. Trans.,' A, vol. 211, pp. 199—251.

† 'Phil. Trans.,' A, 1893, pp. 361—504.

have some easy means of calculating the resistance for any current which may be used.

The change of resistance due to the heating effect of the current may be calculated if the temperature coefficient of resistance and the thermal emissivity are known. Our experiments show that the thermal emissivity is a function of the temperature of the wire, but for a small range of temperature the change is small. If we take the mean emissivity over a range of temperature not greater than 40°, the rise of temperature due to a current may be calculated with a fair degree of accuracy by means of equation (2) which follows, it being assumed, of course, that the rise does not exceed 40°.

If R = resistance of wire, L = its length in centimetres, r = its radius in centimetres, C = current, θ = difference of temperature between the wire and its surroundings when a steady temperature results, h = mean thermal emissivity of the surface in calories per second, J = Joule's equivalent, we have

$$C^2R/J = 2\pi r L h\theta, \tag{1}$$

or

$$\theta = 0{\cdot}038\,C^2R/Lrh. \tag{2}$$

The maximum variation of temperature over a cross-section of a wire has been shown by Dr. A. Russell* to be $\rho\, C^2/16{\cdot}8\, \pi^2 KR^2$ where ρ is the specific resistance and K the thermal conductivity. In the case of the manganin coil used at Hendon, $\rho = 4\times 10^{-5}$ and we may take $K = 0{\cdot}1$, so that for a current of 5 ampères the difference of temperature between the axis and outer skin of the wire was of the order of $0^\circ{\cdot}016$ C. In all that follows, we assume, therefore, that over a cross-section of a wire the temperature is uniform and that no strain is likely to occur because of the temperature gradient.

The usual form of expression for the change of resistance with temperature of a resistance wire is

$$R_t = R_0(1+\alpha t+\beta t^2), \tag{3}$$

where α and β are constants.

When the surroundings of the wire are maintained at a constant temperature and a current C is passed through it, the temperature of the wire is raised, and it follows from (2) that its resistance may be written in the form

$$R_{(C^2)} = R_0\,[1+\alpha\,(aC^2)+\beta\,(aC^2)^2], \tag{4}$$

where $a = 0{\cdot}038\ R_{(C^2)}/Lrh$, and α and β have the same values as before.

If there be some other effect of the current which increases the resistance,

* 'Theory of Electric Cables and Networks,' p. 216.

then an additional term must be added to (4), which would be of the form γC^2, where γ is a constant.

To decide whether or not this is so, we must be able to calculate a, which involves a knowledge of the thermal emissivity. The most direct way of determining the latter is to measure the temperature of the wire while the current is passing through it. Such a measurement could not be made by means of thermo-junctions, as any arrangement of such would not only alter the conditions of cooling near the junction, but it might also be equivalent to increasing the diameter of the wire. One of us has shown that in the case of mercury resistances the temperature and resistance can be measured simultaneously, forming a very convenient mercury-resistance thermometer. The temperature is measured by the expansion of the mercury, and the temperature of a wire can be measured in a very similar way. Our arrangement for measuring the expansion is shown in fig. 1.

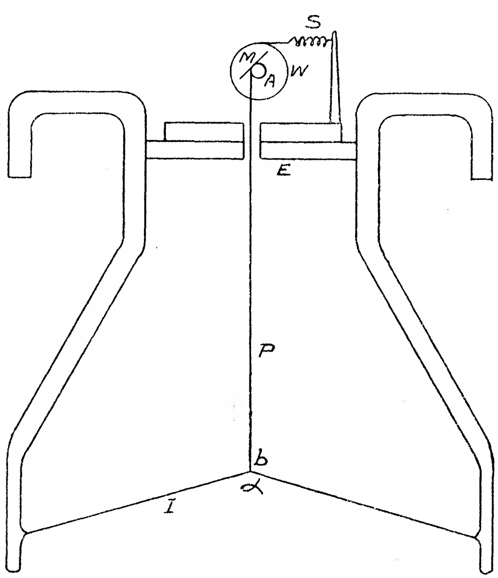


Fig. 1.

The wire experimented with was of iron; it was 12·4 cm. long, 0·02 cm. in diameter, and had been carefully annealed by passing a comparatively large current through it for one hour, the current being then gradually diminished in value so as to ensure slow cooling.

The wire I is connected to two copper leads and is stretched taut by means of a phosphor bronze wire P which passes over an axle A, 4 mm. in diameter. The wire passes round the axle three or four times and is then soldered to it. The wheel W is $1\frac{1}{2}$ cm. in diameter and the spring S ensures

that the wires are always stretched taut. A small mirror M is secured to the axle and reflects a beam of light so as to produce an image at a distance of 1 metre. E is an insulating support of ebonite.

When I increases in length, P is pulled upwards, and the amount by which the point b is raised is optically magnified about 1000 times. In general, the amount by which b is raised is greater than the increase of length of I. If the angle α is nearly 180° the mechanical magnification is considerable. However, we found it more convenient to make α about 150°, in which case the mechanical magnification is about 2. Our estimate of the total magnification was 2000, which means that for the 12·4 cm. length of wire we used, an increase of temperature of 1° should produce a deflection of 2·6 mm.

The apparatus was calibrated by immersing it down to the point b, first in cold and afterwards in hot water. The wire quickly took up the temperature of the water and the remainder of the apparatus was regarded as constant in temperature. The results are given in Table I.

Table I.—Calibration of Temperature Scale.

Temperature of wire.	Difference of temperature.	Scale readings.	Difference of scale readings.
° C.	° C.	mm.	mm.
11 51	40	63 147	84
11 50	39	60 138	78
11 61	50	39 144	105
11 59	48	38 143	105
Sum	177	—	372
	Thus 1° is represented by 372/177 = 2·1 mm.		

The means thus devised of measuring the temperature of a wire during the passage of a current through it rendered it easy to measure its resistance and temperature for any value of the current. The change of resistance with temperature could be determined by a separate series of observations and the validity or invalidity of equation (4) established.

It was of course important to ensure that in any method adopted for measuring the resistance of the wire the other resistances involved did not change, or, if so, changed by known amounts. Dr. E. H. Griffiths, in the paper already referred to, describes several arrangements for measuring the

change of resistance with increasing current. Another arrangement, in which a mercury-resistance thermometer is employed, has also been described by one of us and has been used in making the observations at Hendon. Fortunately, at the National Physical Laboratory such standards of resistance are in use as enable a simple Wheatstone bridge to be used with manganin resistances in three of the arms, of such dimensions as to exclude error from the cause mentioned.

In general, the bridge consisted of two ratio coils of manganin, having nominal values of 10 and 1000 ohms respectively, a heavy current manganin standard of 0·01 ohm, and the wire under observation. All of the resistances were immersed in a bath containing 7 gallons of well-stirred paraffin oil maintained at a constant temperature of 15° or of 20° C. When a balance was obtained (by shunting the 10 or 1000 ohms coils) the same current passed through the 0·01 ohm and the wire (usually of 1 ohm resistance), and since, in all of our measurements, the current density in the 0·01 ohm standard was small, we assumed there to be no appreciable change in the resistance of this standard. The results of our observations show that we were justified in assuming this, but as a check we also used ratio coils of 10 and 10,000 ohms, and a heavy current manganin standard of 0·001 ohm in series with the wire. When the wire had a nominal resistance different from 1 ohm the 10,000 ohms was replaced by a variable resistance.

At the same time as the resistance was measured the temperature of the wire was indicated by the deflections of the spot of light. For values of the current less than 1 ampère no readings of the deflection were taken; for higher currents several readings were taken, as it was found that the errors of observation might amount to three or four degrees. The values of C, C^2, R, and the increase of temperature (θ) of the wire as thus recorded are given in Table II.

Table II.

C.	C^2.	R.	Deflection in mm.	Mean deflection = d.	$\theta = d/2{\cdot}1$.
					° C.
0·2	0·04	0·5760			
0·5	0·25	0·5795			
1·0	1·0	0·5920	19, 16, 16, 12, 14	15	7
1·125	1·27	0·5965	16, 20, 15, 14, 17	16	8
1·5	2·25	0·6127	33, 37, 42, 40, 35	37	18
2·0	4·0	0·6420	54, 61, 50, 64, 56	57	27
2·5	6·25	0·6790	97, 110, 106, 98, 103	103	49
3·0	9·0	0·7255	135, 138, 129, 140, 130	134	64
4·0	16·0	—	247, 241, 250, 247, 234	244	116

Afterwards the change of resistance with temperature of the wire was determined, and these results are plotted on the chart (fig. 2). If, when a current is passed through a wire, the rise of resistance is due to the heating effect only of the current, then for a given resistance the temperature as deduced from the resistance-temperature chart should agree with that obtained from the expansion of the wire. Table III gives the rise of temperature obtained in the two ways.

Table III.

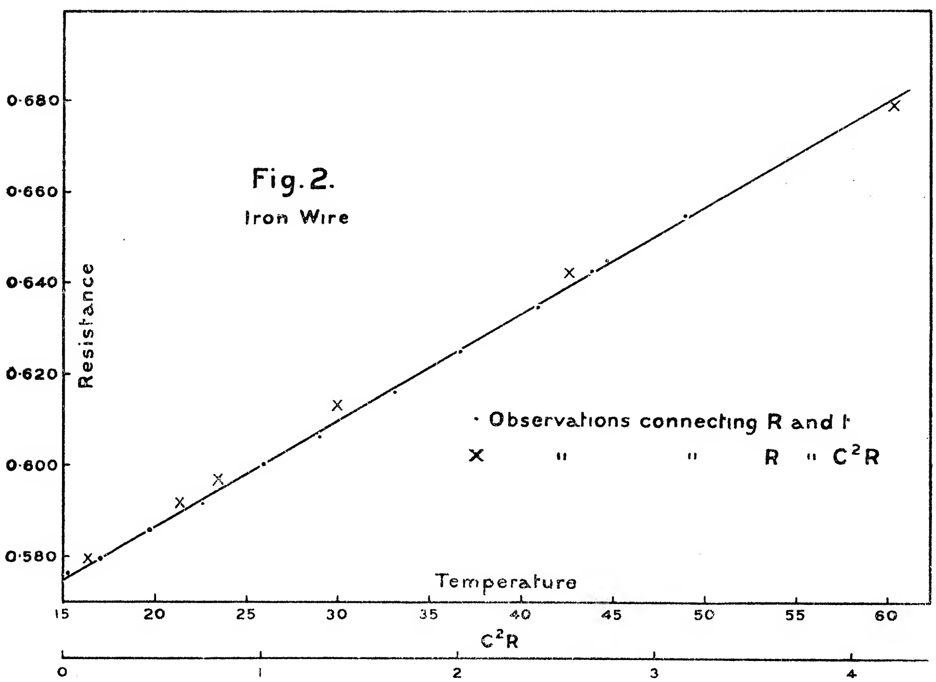
C.	C^2.	R.	θ as given by		Difference.
			Resistance-temperature chart.	Expansion of wire.	
			° C.	° C.	
1·0	1·0	0·5920	7·3	7	−0·3
1·125	1·27	0·5965	9·3	8	−1·3
1·5	2·25	0·6127	16·4	18	+1·6
2·0	4·0	0·6420	29	27	−2
2·5	6·25	0·6790	46	49	+3
3·0	9·0	0·7255	66	64	−2
4·0	16·0	—	—	[116]	
		Sum......	174	173	−1

The values of θ obtained in the two ways being in such close agreement, and the small differences being sometimes positive and sometimes negative, we conclude that the heating effect of the current was the sole cause of the observed increase of resistance. However, we think it desirable to add that when electrical measurements are made with wires not carefully annealed, the passage of a current through them does often produce permanent changes in the resistance. In addition, therefore, to raising the temperature of a wire, a current may so alter the molecular structure as to change, temporarily or permanently, the specific resistance. Very similar effects may be produced by a cycle of temperature, and it is at present difficult to say whether or not the change in the specific resistance is due entirely to the change of temperature of the wire. With well-annealed wires the changes are small.

It is now possible to calculate h, the thermal emissivity of the surface of the wire. For the iron wire experimented with this increases with the temperature, the values being as follows:—

Temp.	h.	Temp.	h.
° C.		° C.	
15 + 7 ·3	0 ·025	15 + 29	0 ·027
9 ·3	0 ·025	46	0 ·028
16 ·4	0 ·026	66	0 ·030

On the diagram (fig. 2) the values of R and C^2R have been plotted in addition to the values of R and temperature, the scales of temperature and C^2R being so chosen that for all practical purposes a single curve connects R and t and R and C^2R. If R and C^2R/h are plotted the resulting curve may be made identical (by suitably choosing the scale of C^2R/h) with that connecting R and t.



The above-described experiments with the iron wire having established the fact that the increase of resistance is substantially due to the increase of temperature, we give the results of some measurements on other wires.

In two of our experiments fuse wires were used. The resistances, lengths, and radii of these were measured, and the melting points were determined in hot glycerine. Portions of the wires were then placed in an electric circuit, and the current gradually increased until they fused. This

was done in air as well as in well-stirred paraffin oil. The results are as follows:—

	Wire No. 1.	Wire No. 2.
Length	27 cm.	18 cm.
Radius	0·01 cm.	0·016 cm.
Resistance	1·0 ohm	0·25 ohm
Melting point	183° C.	226° C.
C in air	1·36 amp.	2·6 amp.
(Mean) *h* in air	0·0015	0·0018
C in oil	6·1 amp.	17·3 amp.
(Mean) *h* in oil	0·031	0·080

In the case of wire No. 2 the melting point is above that of the boiling point (190° C.) of the oil used. This may explain the large mean value of *h* in oil. These experiments show clearly that very large temperature differences between the wire and the oil may exist, even with a well-stirred bath.

Other experiments were made with wires of manganin and of platinum silver, and curves were plotted connecting resistance and temperature and resistance and C^2R. When the scale of temperature was suitably chosen the resistance-temperature curve was in each case practically coincident with the resistance-C^2R curve. The mean value of *h* was of the order 0·03.

The experiments hitherto recorded were made at the National Physical Laboratory. We now come to others, most of which were made at Hendon.

Observations were made on pairs of wires of similar materials, No. 1 of each pair being about 0·03 cm. diameter and No. 2 being about 0·02 cm. in diameter. The wires were bare and immersed in well-stirred paraffin oil, the current being gradually increased from 0·1 to 3 ampères. In the case

Table IV.

Current.	Silver.		Copper.		Aluminium.		Nickel.		Iron.	Platinum.	
	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.	No. 2.	No. 1.	No. 2.
0·1	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
0·2	1·0000	1·0000	1·0002	1·0000	1·0000	1·0000	1·0000	1·0000	1·0001	0·9998	1·0005
0·5	1·0001	1·0000	1·0004	1·0000	1·0002	1·0003	1·0002	1·0009	1·0015	0·9996	1·0022
1·0	1·0005	1·0008	1·0007	1·0010	1·0008	1·0019	1·0020	1·0057	1·0075	1·0001	1·0093
1·5	1·0012	1·0023	1·0013	1·0026	1·0021	1·0044	1·0049	1·0138	1·0174	1·0008	1·0209
2·0	1·0023	1·0047	1·0019	1·0049	1·0038	1·0080	1·0093	1·0253	1·0327	1·0019	1·0379
2·5	1·0036	1·0075	1·0028	1·0078	1·0057	1·0136	1·0148	1·0399	1·0521	1·0034	1·0590
3·0	1·0052	1·0110	1·0040	1·0116	1·0083	1·0190	1·0216	1·0623	1·0781	1·0051	1·0856
3·5			1·0052							1·0073	1·1205
4·0			1·0068							1·0098	1·1578

of the platinum wires and of one copper wire, currents of 3·5 and 4 ampères were also employed. Table IV gives the comparative resistances, the temperature of the bath being practically constant.

For all of the above, curves connecting C^2R and the resistance have been plotted and in all cases practically straight lines result. The temperature coefficients of resistance are given in Table V and these, together with the resistance-C^2R curves enable the relation between θ, the rise of temperature, and C^2R to be found. The mean value of h for the range of temperature θ has been calculated from the equation

$$h = 0{\cdot}038\,C^2R/Lr\theta,$$

where L is the length of wire having a resistance R.

In Table V we give the diameters of the wires, the temperature coefficient of resistance, the resistance per unit length at 20° C., and the mean value of the thermal emissivity.

Table V.

Wire.	Diameter.	Approximate temperature coefficient of resistance at 20°.	Resistance per centimetre at 20°.	*h*.
Silver, No. 1	0·0305	0·00370	0·00229	0·037
„ No. 2	0·0200	0·00352	0·00523	0·058
Copper, No. 1	0·0303	0·00390	0·00243	0·054
„ No. 2	0·0199	0·00392	0·00564	0·066
Aluminium, No. 1	0·0302	0·00402	0·00399	0·044
„ No. 2	0·0200	0·00407	0·00898	0·067
Nickel, No. 1	0·0303	0·00446	0·01283	0·061
„ No. 2	0·0200	0·00445	0·02941	0·076
Iron, No. 2	0·0200	0·00400	0·0465	0·088
Platinum, No. 1	0·0505	0·00394	0·00484	0·051
„ No. 2	0·0173	0·00394	0·0417	0·082

It will be observed that h is greater for the small wires than for the larger ones; we have already seen that it increases slightly with the temperature. The mean value 0·062 is about twice that observed at Teddington with wires of iron, manganin, and platinum silver. The cause of this may be partly due to a difference in the paraffin oils, but we believe it to be principally due to the fact that much more vigorous stirring is employed at Hendon.

The importance of uniform stirring must not be overlooked. Dr. E. H. Griffiths tried the effect of variations, and found, as would be expected, that the more perfect the stirring the less the wire became heated, but within the limits of stirring employed in his experiments on the mechanical equivalent of heat, no correction was thought to be necessary.

In the case of copper wire No. 1, the following results were obtained with a current of 3 ampères:—

	Resistance.
Normal stirring	1·0038
Very vigorous stirring	1·0027
No stirring	1·0173

These figures seem to show that the normal stirring employed at Hendon was vigorous, and that the value of h resulting from such stirring is as great as would be likely to be realised in ordinary practice with bare wires immersed in paraffin oils.

In the case of wires covered with silk and shellac, the heating effect is greater than with bare wires, and is best found experimentally by measuring the variation of resistance with temperature and of resistance with current.

We now briefly indicate the importance for precise electrical measurements of the heating effect of the current.

For such electrical measurements as those involved in the determination of the mechanical equivalent of heat, it is clearly of importance, especially when coils with large temperature coefficients of resistance are used.

Dr. E. H. Griffiths, who employed a platinum wire for the heating, allowed for this change of resistance, and pointed out the importance of determining the resistance for various currents.

In Dr. Barnes' determination of the mechanical equivalent the current was measured by measuring the potential difference on a coil of platinum silver through which the current flowed. This coil was immersed in well-stirred oil, and Dr. Barnes believed the increase of temperature of the wire to be less than 0·1° C. While this may be possible, it is of interest to point out that in Dr. Barnes' experiments the maximum current which passed through one platinum-silver wire was 1 ampère. Each wire was about 100 cm. long, 0·02 cm. radius, and 4 ohms in resistance. If these values are substituted for C, L, r, and R in equation (2), we find that θ, the rise of temperature, is equal to $0·076/h$. If the cooling effect of the stirring was the same as at the National Physical Laboratory (*i.e.*, if $h = 0·03$), the value of θ is $2^{\circ}·5$, equivalent to an increase of the resistance of 6 parts in 10,000. If, on the other hand, the cooling effect of the stirring corresponded to the most vigorous stirring set up at Hendon, the rise of temperature would be $0^{\circ}·95$ C., equivalent to 2·4 parts in 10,000 of the resistance.

For the comparison of standards of resistance the heating effect of the current is sometimes overlooked, especially when a comparison of methods is made.

For instance, it is generally thought that in bridge methods for the

comparison of resistance much larger currents can be used than in the potentiometer method, because in the bridge methods the currents pass through the resistances for a few seconds only. In general, however, this is a mistake, for the temperature of a resistance coil in oil rises to within a tenth of a degree of the maximum temperature within a few seconds of the circuit being closed.

If the maximum increase of temperature of the wire is θ_1 and if θ is the increase of temperature t seconds after making the circuit, we have

$$\log_e \theta_1 - \log_e(\theta_1 - \theta) = 2ht/\Delta r \mathrm{S}, \tag{5}$$

where Δ is the density of the material of the wire, and S is the specific heat.

Experiments were made on a manganin wire 0·6 mm. in diameter, and the resulting values of h were 0·00115 in air and 0·021 in oil. The value of Δ S was about 1·0. For a current of 1·5 ampère the maximum elevation of temperature of the wire in air was 38°, the actual temperature being $20°+38° = 58°$. By means of equation (5) we have calculated the time in seconds, after closing the circuit, corresponding to various values of $\theta_1 - \theta$. The times and temperatures of the wire are given in Columns 1 and 2 of Table VI. As a check on these values we measured the change of resistance of the coil (by observing the galvanometer deflection) during the first minute of the passage of the current of 1·5 ampère, and from the resistance values we estimated the temperature with the aid of a resistance-temperature chart. These times and temperatures are recorded as "observed" values, and are given in Columns 3 and 4.

Table VI.

Current = 1·5 ampère. Maximum temperature = 58° C.

Calculated.		Observed.	
Time.	Temp.	Time.	Temp.
secs.	° C.	secs.	° C.
15	46	12	46
18	48·5	20	48·5
20	50	24	50
25	52·5	27	52·5
28	53·5	33	53·5
35	55·5	37	55·5
47	57	50	57

In oil a current of about 6·4 ampères was required to raise the temperature to 58°, but since $2h/\Delta r\mathrm{S}$ is about twenty times the value in air, the rate of increase of temperature is much greater. The final temperature of 58° is reached within 0°·1 in less than 5 seconds after closing the circuit.

APPENDIX.—By W. R. BOUSFIELD, K.C.

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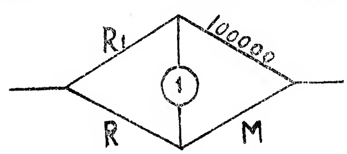
It is a matter of interest to determine the exact relation of the actual effect to the radius of the wire, but the experiments previously described were not sufficiently comparable *inter se* to enable this to be done. Differences in stirring produced notable differences in the results, and even differences in the winding of the wire on the mica plate were material. To bring these variable elements under control, the wire was soldered to two heavy copper electrodes and lightly stretched longitudinally in a glass tube, through which flowed a current of cooling water. Owing to the velocity of the water and to the fact that the wire was led into and out of the main tube by side tubes at right angles, the flow was necessarily turbulent. Some transverse vibration of the wire produced by the water current also tended to prevent mere gliding flow. The diameter of the tube was 0·97 cm. and the rate of flow (except in one experiment) was 100 c.c. per second, giving a mean velocity in the direction of the length of the tube of about 135 cm./sec. In the subjoined experiments the wires used were the soft platinum wire of 0·0505 cm. diameter which had been used for a previous experiment in the stirred oil bath, and a series of smaller wires drawn from a portion of the same wire so as to be completely comparable as regards material.* The figures for the experiment on the first wire of the series are given below. In the bridge arrangement, R, the resistance to be measured, was in series with a mercury



thermometer resistance M, and the other arms of the bridge consisted of manganin resistances, one of 100,000 ohms and the other, R_1, adjustable. A current of 5 ampères was passed through the wire for an hour before taking the measurements. The second column in the table records the value of $C^2 \times 25$ as determined by an ampère balance in the circuit, the accuracy of which could be relied upon within 1 in 5,000. The temperature of the inflow water during the experiment was 20·85° C. The length of the wire was 24·5 cm. The resistance of the column of tap water in the tube between the thick copper electrodes (measured by means of the Kohlrausch

* Our best thanks are due to Messrs. Johnson and Matthey, who were kind enough to give special attention to the drawing of the various wires required for the purposes of the investigation.

wheel-bridge and telephone before soldering in the wire) was 82,000 ohms, *i.e.*, about 600,000 times the resistance of the wire. The correction of R for the conductivity of the water is therefore insensible.

Platinum Wire No. 1. Diameter, 0·0505 cm. Water flow, 100 c.c. per second.

Approximate current.	$C^2 \times 25$.	R observed.	R calculated.	Difference.
0·6	10	0·13546	0·13547	+1
0·9	20	0·13546	0·13547	+1
1·3	40	0·13548	0·13548	±
1·8	80	0·13550	0·13550	±
2·5	160	0·13554	0·13554	±
3·6	320	0·13562	0·13562	±
5·1	640	0·13578	0·13578	±

The calculated values of R which are given in the table are from the expression

$$R = 0{\cdot}13546 + 0{\cdot}0000005\, C^2 \times 25.$$

This gives for α in the expression $R/R_0 = 1 + \alpha C^2$ the value $\alpha = 0{\cdot}000092$, which may be compared with the value found in the oil bath with "normal" stirring, which was $\alpha = 0{\cdot}0014$.

Hence by this arrangement, with a water flow of 100 c.c. per second, the effect is reduced to one-fifteenth of its former magnitude. If a platinum wire of 0·05 cm. diameter and 1 metre in length, having a resistance of about half an ohm, be used as a standard resistance for carrying currents up to 5 ampères, mounted as above described in a tube of about 1 cm. diameter, with a regulated flow of water of about 100 c.c. per second, it is therefore easy to apply the necessary correction, so as to obtain an accuracy of 1 in 50,000 on the resistance measured, and the error introduced by the conductivity of the tap water will be insensible.

In order to test the influence of the nature of the surface of the wire on the result, the same wire was coated with platinum black electrolytically, and afterwards submitted to a white heat in the flame of a Bunsen burner, whereby the platinum wire acquired a dull grey matt surface. The result was a small further reduction of α from 0·000092 to 0·000084.

In order to test the influence of water velocity on the result, the wire was submitted to a continuous constant current of a little over 5 ampères ($C^2 = 640/25$), and the bridge measurements were taken with the water flow varying from 100 c.c. per second to 1·37 c.c. per second. The results of this series of experiments are given in the subjoined table. The inflow temperature of the water was 21·3° C. The outflow temperature was

sensibly the same as that of inflow for the first four or five measurements. For the last the outflow was about 0·6° higher than the inflow.

W. Water flow per second, in c.c.	R. Observed.	ΔR.		Difference.
		Observed.	Calculated.	
99·0	0·14026	12	12	±
93·0	0·14027	13	12	− 1
87·0	0·14028	14	13	− 1
74·5	0·14031	17	15	− 2
70·0	0·14035	21	16	− 5
55·0	0·14040	26	21	− 5
49·0	0·14044	30	23	− 7
40·0	0·14047	33	28	− 5
30·0	0·14056	42	36	− 6
26·0	0·14061	47	41	− 6
17·3	0·14079	65	58	− 7
12·3	0·14090	76	77	+ 1
8·2	0·14116	102	104	+ 2
4·7	0·14182	168	150	−18
1·37	0·14271	257	257	±

The calculated values of ΔR are obtained from the expression

$$R = 0{\cdot}14014 + \frac{0{\cdot}012}{W + 3{\cdot}3}.$$

This expression gives the value of R with infinite water flow as 0·14014. The "observed" values of ΔR are obtained by subtracting 0·14014 from the observed values of R.

The series of values of ΔR illustrates well the great importance of effective and regular stirring in securing a reliable correction for the effect. Even with the very small flow of 1·37 c.c. per second, the mean temperature of the cooling water is only 0·3° higher than in the first experiment, but the value of ΔR is 20 times as great, and the increase in the resistance is nearly 2 per cent.

With slow velocity of flow the turbulence ceases, and is replaced by a slow, gliding motion of the water along the wire. It is to this that the increased resistance is to be attributed, and not to the slight increase of temperature of the cooling water.

To determine the actual law of the effect as depending on the radius of the wire, four other wires were taken, drawn from the same material, and the resistances measured with varying currents and a uniform water flow of 100 c.c. per second. The results are given below; the resistance of each wire measured with an indefinitely small current being taken as unity. All these series of values satisfy the equation

$$R/R_0 = 1 + \alpha C^2.$$

Experiment on Five Platinum Wires with uniform Water Flow of 100 c.c. per second.

$C^2 \times 25$	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
10	1·0000	1·0001	1·0003	1·0006	1·0014
20	1·0000	1·0001	1·0003	1·0011	1·0029
40	1·0001	—	1·0010	1·0023	1·0054
60	—	—	—	—	1·0085
80	1·0003	—	—	1·0045	1·0118
100	—	1·0008	1·0026	—	1·0143
150	—	—	1·0038	—	1·0212
160	1·0006	—	—	1·0090	—
225	—	1·0020	1·0062	1·0123	1·0313
300	—	—	1·0081	1·0169	—
320	1·0012	—	—	—	—
400	—	1·0036	1·0109	1·0224	—
450	—	—	1·0135	—	—
500	—	—	—	1·0272	—
640	1·0024	1·0058	1·0175	1·0345	—

The values of α, together with the diameters d of the wires, are given in the following table. The values for the diameters are each the mean of 10 measurements.

No. of wire.	d.	$\alpha \times 10^2$.	Log $(d \times 10^2)$.	Log $(\alpha \times 10^5)$.	$\alpha \times 10^2$ calculated.
1	0·0505	0·0092	0·703	0·962	0·0092
2	0·0352	0·0225	0·547	1·352	0·0225
3	0·0227	0·0675	0·356	1·826	0·0674
4	0·0169	0·140	0·228	2·146	0·141
5	0·0117	0·355	0·068	2·550	0·354

If in a diagram there are set out the values of $\log(d \times 10^2)$ as ordinates, and the values of $\log(\alpha \times 10^5)$ as abscissæ, the result is a straight line, which gives the relation

$$\log \alpha = \bar{8}{\cdot}719 - \tfrac{5}{2} \log d.$$

The values of α calculated from this expression are given in the last column of the table, and they correspond very accurately with the observed values. Hence, for a series of wires of similar material, similar surface, and with the same water flow, we have

$$\alpha = r^{-\frac{5}{2}} \times \text{a constant.}$$

If w be the resistance of the wire per unit of length and ρ the specific resistance, we have $w = \rho/\pi r^2$, and therefore

$$\alpha = r^{-\frac{1}{2}} w \times \text{a constant.}$$

Furthermore, assuming that the whole effect is due to rise of temperature of the wire, and within the range for which the temperature-resistance curve may be taken as a straight line, we have

$$R/R_0 = 1+\alpha C^2, \quad R/R_0 = 1+\beta\theta,$$

where β is the ordinary temperature coefficient. Also by equation (1)

$$h = \frac{C^2}{\theta} \times \frac{R}{L} \times \frac{1}{2\pi J r};$$

and since $C^2/\theta = \beta/\alpha$ and $R/L = w$, we have

$$h = \beta w/2\pi J r\alpha, \quad \text{or} \quad h = \beta/2\pi J r^{\frac{1}{2}}.$$

Thus it follows for a series of wires of the same material that

$$h = r^{-\frac{1}{2}} \times \text{a constant}.$$

It thus appears that the "emissivity" of a round wire in contact with a liquid cooling agent is proportional to the inverse square root of the radius, when the other conditions of convective action of the cooling agent at the surface of the wire are kept constant.

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